# CONCEPT QUICKSTART – Increasing and Decreasing Functions

**Unit: Unit 6: Application of Derivatives**

**Subject:For CBSE Class 12 Mathematics**

**SECTION 1: UNDERSTANDING THE CONCEPT**

Monotonicity—the study of whether a function is increasing or decreasing—is a strategic cornerstone of calculus. Rather than laboriously plotting individual points to understand a curve's shape, we use the derivative as a **behavioral compass**. By calculating the sign of the first derivative, we gain an immediate, high-level view of a function's trajectory. This allows you to visualize whether a graph is climbing, falling, or resting at any given interval, turning abstract equations into predictable, visible paths. Let's break this down into tiny, manageable steps so you can master this with confidence.

**1.1 What Is Increasing and Decreasing Functions?** The "Big Idea" of this topic is to use the derivative as a practical tool to reveal which regions of a function grow or shrink and where it reaches its turning points. In essence, the derivative identifies the **monotonicity** or the direction of change within specific intervals of the function's domain.

**Expert Insight**: In mathematics, we always read a graph from left to right (increasing values of x). If the height of the graph (the y-value) moves upward as you move right, the function is **increasing**; if it moves downward, it is **decreasing**. A common misconception among students is that only straight lines (linear functions) can be increasing or decreasing. In reality, complex curves, parabolas, and trigonometric waves all exhibit these behaviors over specific intervals.

**1.2 Why It Matters** Understanding monotonicity is vital for identifying **turning points**, which are the transition zones between climbing and falling. Beyond the classroom, this concept is a fundamental building block for **profit modeling** in business—where we seek intervals of increasing revenue—and in **engineering** and **physics**, where it helps determine the stability and optimization of systems.

**1.3 Prior Learning Connection** To succeed in this topic, you must have a "Survival Toolset" from Chapter 5 (**Continuity and Differentiability**), specifically:

- **Mastery of Chapter 5 Differentiation Rules**: You must be able to calculate f'(x) quickly using the Power, Product, and Quotient rules.
- **The Chain Rule**: This is absolutely essential for handling composite functions (e.g., sin 3x).
- **Basic Factorization**: This is your best friend when finding critical points where the derivative equals zero.

**1.4 Core Definitions** The following formal definitions and theorems from the NCERT textbook provide the "rules of the game" for interval analysis:

**[Definition 1(i): Increasing]**

- NCERT Reference: Section 6.3 / Page 152
- Definition: f is increasing on I if $x_1 < x_2$ in I $\Longrightarrow f(x_1) \leq f(x_2)$ for all $x_1, x_2 \in I$.
- Used In: Problem Type F1

**[Definition 1(ii): Decreasing]**

- NCERT Reference: Section 6.3 / Page 152
- Definition: f is decreasing on I if $x_1 < x_2$ in I $\Longrightarrow f(x_1) \geq f(x_2)$ for all $x_1, x_2 \in I$.
- Used In: Problem Type F1

**[Definition 1(iv): Strictly Increasing]**

- NCERT Reference: Section 6.3 / Page 153
- Definition: f is strictly increasing on I if $x_1 < x_2$ in I $\Longrightarrow f(x_1) < f(x_2)$ for all $x_1, x_2 \in I$.
- Used In: Problem Type F1 and F2

**[Definition 1(v): Strictly Decreasing]**

- NCERT Reference: Section 6.3 / Page 153
- Definition: f is strictly decreasing on I if $x_1 < x_2$ in I $\Longrightarrow f(x_1) > f(x_2)$ for all $x_1, x_2 \in I$.
- Used In: Problem Type F1 and F2

**[Theorem 1: First Derivative Test for Monotonicity]**

- NCERT Reference: Section 6.3 / Page 153
- Definition: Let f be continuous on [a, b] and differentiable on the open interval (a, b). Then: (a) f is increasing in [a, b] if $f'(x) > 0$ for each $x \in (a, b)$. (b) f is decreasing in [a, b] if $f'(x) < 0$ for each $x \in (a, b)$. (c) f is a constant function in [a, b] if $f'(x) = 0$ for each $x \in (a, b)$.
- Used In: Problem Type F1 and F2

These formal definitions provide the logical structure we need to solve NCERT exercises. By mastering this technical language, you ensure that your descriptive answers meet the high standards of the CBSE marking scheme.

**SECTION 2: WHAT NCERT SAYS**

The NCERT textbook is the "Gold Standard" for CBSE examinations because the board's marking schemes are derived directly from its definitions and proofs. When answering descriptive questions, using the exact language found in NCERT—such as specifying **continuity on a closed interval** and **differentiability on an open interval**—is the "So What?" layer that secures full marks. It shows the examiner you aren't just calculating; you are reasoning mathematically.

**2.1 Key Statements**

1. **Theorem 1 Prerequisites**: For the **First Derivative Test** to apply, a function must be continuous on the closed interval [a, b] and differentiable on the open interval (a, b).
2. **The Constant Condition**: If f'(x) = 0 for every x in an interval (a, b), the function is constant; it neither climbs nor falls.
3. **Strict Monotonicity**: A function is **strictly increasing** if f'(x) is strictly greater than 0, and **strictly decreasing** if f'(x) is strictly less than 0 across the interval.
4. **The "Neither" Case**: Be careful! Many functions are neither increasing nor decreasing over their entire domain. For example, f(x) = sin x is neither increasing nor decreasing in (0, 2π) because it changes direction (NCERT Example 9c).
5. **Critical Points**: We look for **critical points** where f'(x) = 0 or f'(x) is undefined. These are the "gateways" where the behavior of the function might change.

**2.2 Examples and Exercises** Mastering these "Anchor Examples" is a "must-solve" strategy for your exam preparation:

- **Example 10 (Page 155)**: Shows how to find intervals for a polynomial ($x^2 - 4x + 6$). This is the classic introduction to partitioning the number line.
- **Example 12 (Page 156)**: Demonstrates trigonometric interval finding (sin 3x). This is a "must-solve" because it requires careful handling of the angle multiplier.
- **Example 13 (Page 156/157)**: Analyzes sin x + cos x. This is critical for learning how to handle mixed trigonometric terms using identities.

**Exercise 6.2 Ranges**:

- **Foundational (Questions 1-3)**: Focused on **verification**—proving a function is increasing or decreasing on a given set like $\mathbb{R}$.
- **Critical (Questions 4-6, 12-13)**: Focused on **discovery**—finding the specific intervals where monotonicity changes. These are high-weightage patterns that frequently appear in exams.

**SECTION 3: PROBLEM-SOLVING AND MEMORY**

Success in CBSE Mathematics relies on a **Pattern Recognition** mindset. The exam rarely introduces entirely new problems; instead, it presents variations of specific "Families." In this landscape, the **sign diagram** is your most powerful tool, allowing you to visually organize where a function's derivative changes its "mood." Don't panic if the algebra looks messy at first—we will break it down step-by-step.

### 3.1 Problem Types

**Problem Type: Family F1 (Verify Monotonicity)**

- **Structural Goal**: Prove that a function is increasing or decreasing on a pre-specified interval.
- **Recognition Cues**:
  - **Surface**: Look for "Show that..." or "Prove that...".
  - **Structural**: Often involves linear, exponential ($e^{2x}$), or simple polynomial functions.
- **What You're Really Doing**: You are demonstrating that f'(x) maintains a constant sign (always positive or always negative) throughout the requested range.
- **NCERT References**: Example 7, Exercise 6.2 Q1.
- **Confusable Types**: Often mixed up with Family F2. Remember: F1 gives you the interval; F2 asks you to find it.

**Problem Type: Family F2 (Find Intervals)**

- **Structural Goal**: Discover the exact regions where a function climbs, falls, or stays constant.
- **Recognition Cues**:
  - **Surface**: Keywords like "Find intervals..." or "On which interval is it strictly decreasing?".
  - **Structural**: Look for cubic polynomials or trigonometric functions.
- **What You're Really Doing**: You are partitioning the number line into **disjoint intervals** using critical points and testing each region's sign.
- **NCERT References**: Example 10, Example 11.
- **Confusable Types**: Can be confused with finding Maxima/Minima (Topic 4). While related, Family F2 ends with intervals, not points.

### 3.2 Step-by-Step Methods

**Type: Finding Intervals (Solution Method for Family F2)**

- **Pre-Check (Critical Conditions)**: Identify the domain of the function first! For example, log x is only defined for $x > 0$. Ensure the function is continuous.
- **Core Steps**:
  - **Step 1: Compute Derivative f'(x) (Setup)**: Differentiate accurately using Chapter 5 rules. Take your time here; a mistake in f' ruins the whole problem!
  - **Step 2: Find Critical Points (Transform)**: Set $f'(x) = 0$ and solve for x. These points are where the graph might turn.
  - **Step 3: Partition Number Line (Apply)**: Place these x-values on a number line to create **disjoint intervals** (e.g., $(-\infty, 2)$ and $(2, \infty)$).
  - **Step 4: Test Sign in Each Interval (Analyze)**: Pick an easy test point in each interval (like 0 or 1), plug it into **f'(x)**, and check if it is (+) or (−).
- **Trigonometric Variants**: When dealing with sin(nx), remember that $3x = \pi/2$ implies $x = \pi/6$. Always verify if your x-values fall within the range specified in the question (e.g., $[0, \pi/2]$).
- **When NOT to Use**: Do not use this if the function is not differentiable or if you are only asked to verify a specific point.

**3.3 How to Write Answers**

**Answer Template (The CBSE Frame)**:

- **L1 (Setup)**: State the function f(x) and its domain.
- **L2 (Derivative)**: Show the differentiated form f'(x) clearly.
- **L3 (Critical Points)**: Write "For critical points, $f'(x) = 0$" and solve for x.
- **L4 (Interval Analysis)**: List the **disjoint intervals** and show the sign of f'(x) in each.
- **L5 (Conclusion)**: State the final intervals using the correct notation.

**Essential Phrases**:

- "By Theorem 1, f is increasing in..."
- "$f'(x) > 0$ for all x in the interval..."
- "The points $x = a$ and $x = b$ divide the real line into disjoint intervals..."

**General Rules**:

- Use proper interval notation: Use parentheses ( ) for open intervals and brackets [ ] for closed intervals as per Theorem 1.
- Show your factorization clearly to avoid "silly" marks deductions.

### 3.4 Common Mistakes

- **Pitfall 1: The Test Point Trap**
  - **Category**: Algebra/Logic
  - **Wrong**: Plugging the test point into the original f(x) instead of the derivative f'(x).
  - ✓ **Fix**: Always test signs using f'(x). The original function tells you "where" you are; the derivative tells you "where you are going."
- **Pitfall 2: Strict vs. Non-Strict Confusion**
  - **Category**: Logic
  - **Wrong**: Writing "strictly increasing" when the derivative could be zero at some points in the interval.
  - ✓ **Fix**: Use $f'(x) > 0$ for **strictly increasing** and $f'(x) \geq 0$ for **increasing**. Check what the question specifically asks for!
- **Pitfall 3: Non-Differentiability Ignored**
  - **Category**: Logic
  - **Wrong**: Forgetting that points where f'(x) is undefined are also critical points (e.g., $x = 0$ for $f(x) = |x|$).
  - ✓ **Fix**: Check for both $f'(x) = 0$ and points where f'(x) does not exist.
- **Pitfall 4: Endpoint Confusion**
  - **Category**: Logic
  - **Wrong**: Including an endpoint where the function is not defined or continuous.
  - ✓ **Fix**: Always refer back to the function's domain before finalizing your intervals.

**3.5 Exam Strategy** For a **Mastery Approach**, begin with polynomial verifications (Family F1). Once confident, move to finding intervals for polynomials, and finally tackle **trigonometric intervals**.

- **3-Mark Patterns**: Typically involve finding intervals for a cubic polynomial.
- **5-Mark Patterns**: These often appear in **Section D** and involve complex trigonometric combinations (like Example 13) or proving monotonicity for a logarithmic/exponential mix. Trigonometric interval questions are the most common high-weightage types.

### 3.6 Topic Connections

- **Prerequisites**: Mastery of Chapter 5 differentiation is non-negotiable. If you struggle with the Chain Rule, revisit Chapter 5 before attempting trig intervals.
- **Forward Links**: This topic leads directly to **Maxima and Minima (Topic 4)**. If the derivative changes from (+) to (−) at a point, you have found a **Local Maximum**.

**3.7 Revision Summary**

1. **$f'(x) > 0$** means the function is **strictly increasing**.
2. **$f'(x) < 0$** means the function is **strictly decreasing**.
3. **Critical Points** occur where **$f'(x) = 0$** or is undefined.
4. Always read graphs from **left to right**.
5. Use a **Sign Diagram** to keep test results organized and visual.
6. **Theorem 1** requires continuity on [a, b] and differentiability on (a, b).
7. Check the **natural domain** (e.g., $\log x$ requires $x > 0$) before you start.
8. Trig functions need careful **angle division** ($3x = \pi/2 \implies x = \pi/6$).

**Memory Aid: The DSLT Checklist**

- **Differentiate**: Get $f'(x)$ accurately.
- **Solve**: Set $f'(x) = 0$ to find critical points.
- **Line**: Draw the number line and mark the points.
- **Test**: Check the sign (+) or (−) in each interval gap.
- **Conclude**: Write the final intervals using **disjoint intervals** terminology.