CONCEPT QUICKSTART – Types of Relations

Unit: Unit1: Relations and Functions

Subject: For CBSE Class 12 Mathematics

SECTION 1: UNDERSTANDING THE CONCEPT

This section builds the foundational understanding of what different types of relations are. We will explore why these classifications are important and how they connect to concepts you have already mastered in Class 11, setting a strong base for the rest of the chapter.

1.1 What Is This Topic About?

This topic is about classifying relations based on specific properties or "rules" they follow. The three main properties we will study are **reflexivity**, **symmetry**, and **transitivity**. These properties describe how elements within a set are connected to themselves and to each other under a given relation. A common misunderstanding is that a relation must satisfy all these properties at once. This is not true. A relation can have any combination of these properties—it might be only symmetric, or it might be reflexive and transitive but not symmetric, or it might be all three.

1.2 Why It Matters

Classifying relations helps us understand the underlying structure of sets and mathematical systems. The most significant concept that emerges from this is the **Equivalence Relation**—a relation that is reflexive, symmetric, and transitive. This is a powerful idea because it provides a mathematical way to group similar objects together into distinct categories called "equivalence classes." This idea of partitioning a large set into smaller, disjoint groups is a fundamental tool used in many areas of higher-level mathematics, such as how the set of all integers can be partitioned perfectly into the set of even integers and the set of odd integers.

1.3 Prior Learning Connection

Think of this topic as the next level up from what you learned in Class 11. Make sure you are comfortable with:

- **Set Theory:** A relation is defined on a set, so you must be comfortable with set notation and concepts like an element belonging to a set (e.g., $a \in A$).
- **Cartesian Product ($A \times A$):** A relation is fundamentally a subset of the Cartesian product of a set with itself ($R \subset A \times A$).
- **Ordered Pairs:** Relations are defined as sets of ordered pairs, where the order of elements in the pair (a, b) matters.

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1.4 Core Definitions

These are the formal definitions from your NCERT textbook that you must know.

- **Empty Relation**
  - **NCERT Reference:** Page 2, Definition 1
  - **Definition:** A relation R in a set A is called an empty relation if no element of A is related to any element of A, i.e., $R = \emptyset \subset A \times A$.
  - **Used In:** Problem Type: Identifying trivial relations.
- **Universal Relation**
  - **NCERT Reference:** Page 2, Definition 2
  - **Definition:** A relation R in a set A is called a universal relation if each element of A is related to every element of A, i.e., $R = A \times A$.
  - **Used In:** Problem Type: Identifying trivial relations.
- **Reflexive Relation**
  - **NCERT Reference:** Page 2, Definition 3
  - **Definition:** A relation R in a set A is called reflexive if $(a, a) \in R$, for every $a \in A$.
  - **Used In:** Problem Types RF (Reflexive Check), EQ (Equivalence Proof).
- **Symmetric Relation**
  - **NCERT Reference:** Page 2, Definition 3
  - **Definition:** A relation R in a set A is called symmetric if $(a_1, a_2) \in R$ implies that $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$.
  - **Used In:** Problem Types SM (Symmetric Check), EQ (Equivalence Proof).
- **Transitive Relation**
  - **NCERT Reference:** Page 2, Definition 3
  - **Definition:** A relation R in a set A is called transitive if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R$ implies that $(a_1, a_3) \in R$, for all $a_1, a_2, a_3 \in A$.
  - **Used In:** Problem Types TR (Transitive Check), EQ (Equivalence Proof).
- **Equivalence Relation**
  - **NCERT Reference:** Page 3, Definition 4

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- **Definition:** A relation R in a set A is said to be an equivalence relation if R is reflexive, symmetric, and transitive.
- **Used In:** Problem Type EQ (Equivalence Proof).

SECTION 2: WHAT NCERT SAYS

This section summarizes the key ideas, examples, and exercises presented directly in your NCERT textbook. Focusing on these points will ensure your learning is perfectly aligned with the official CBSE curriculum.

2.1 Key Statements

Here are the most important takeaways from the NCERT text, paraphrased for clarity.

1. The empty relation and the universal relation are sometimes referred to as **trivial relations**.
2. An **equivalence relation** is a particularly important type that satisfies all three conditions: reflexivity, symmetry, and transitivity.
3. An equivalence relation R on a set X divides the set into mutually disjoint subsets, which are called **equivalence classes** or partitions.
4. Within any single equivalence class, all elements are related to each other.
5. No element of one equivalence class is related to any element of a different equivalence class.

2.2 Examples and Exercises

These worked examples from the NCERT textbook are essential for understanding the concepts.

- **Worked Example 2 (Page 3):**
  - **What it shows:** Proves that the relation "is congruent to" on a set of triangles is an equivalence relation.
  - **Why it's important:** It is a classic, intuitive example of an equivalence relation from geometry, clearly demonstrating the reflexive, symmetric, and transitive properties.
- **Worked Example 3 (Page 3):**
  - **What it shows:** Proves that the relation "is perpendicular to" on a set of lines is symmetric, but **neither** reflexive **nor** transitive.

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- Why it's important: This is a crucial counterexample that demonstrates how a relation can satisfy one property (symmetry) but fail the other two, preventing it from being an equivalence relation.

- **Worked Example 5 (Page 3):**
    - **What it shows:** Proves that the relation R = {(a, b) : 2 divides a – b} on the set of integers is an equivalence relation.
    - **Why it's important:** This example introduces equivalence relations on numbers and provides the first clear illustration of **equivalence classes**: the set of all even integers [0] and the set of all odd integers [1].
- **NCERT Exercises:**
    - **Exercise Set:** 1.1
    - **Question Numbers:** 1–16

SECTION 3: PROBLEM-SOLVING AND MEMORY

This section focuses on practical application. It provides structured methods to identify and solve common problem types, write answers correctly for your exams, avoid frequent mistakes, and revise the topic effectively.

3.1 Problem Types

In your exam, questions about types of relations will almost always ask you to do one of two things:

- **Problem Type: Checking Relation Properties (Reflexive, Symmetric, Transitive)**
    - **Structural Goal:** To determine if a given relation R on a set A satisfies one or more of the three main properties.
    - **Recognition Cues:** Look for keywords like "Determine whether the relation is...", "Check whether the relation is reflexive, symmetric or transitive."
    - **What You're Really Doing:** You are systematically testing the relation against the formal definition of each property. For reflexivity, you check if every element is related to itself. For symmetry, you check if the reverse of every pair is also in the relation. For transitivity, you check if chains of relations are completed.
    - **NCERT References:** Examples 3, 4 | Exercises 1.1 (Q1-6).
    - **Confusable Types:** Proving a full equivalence relation (this is just one part of that process).

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- **Problem Type: Proving a Relation is an Equivalence Relation**
  - **Structural Goal:** To write a formal proof demonstrating that a given relation R satisfies all three properties: reflexivity, symmetry, AND transitivity.
  - **Recognition Cues:** Look for keywords like "Show that the relation R is an equivalence relation."
  - **What You're Really Doing:** You are executing three mini-proofs in a specific sequence and then writing a final conclusion.
  - **NCERT References:** Examples 2, 5, 6 | Exercises 1.1 (Q7-14).
  - **Confusable Types:** Merely checking the properties without structuring it as a formal proof.

3.2 Step-by-Step Methods

Here is a reliable method for the most important problem type you will face.

- **Type: Proving an Equivalence Relation: Solution Method**
  - **Pre-Check:** Clearly identify the set A the relation is defined on and the exact condition that must be met for an ordered pair (a, b) to be in R.
  - **Core Steps:**
    - **Step 1: Prove Reflexivity** (Setup)
      - Start by stating: "For any $a \in A$...".
      - Apply the relation's condition to the pair (a, a) and show that it holds true.
      - Conclude: "Therefore, $(a, a) \in R$, and R is reflexive."
    - **Step 2: Prove Symmetry** (Apply)
      - Start by assuming a pair is in the relation: "Let $(a, b) \in R$." This means the condition is true for (a, b).
      - Use algebra or logic to show that the condition must also be true for the pair (b, a).
      - Conclude: "Therefore, $(b, a) \in R$, and R is symmetric."
    - **Step 3: Prove Transitivity** (Apply)
      - Start by assuming two linked pairs are in the relation: "Let $(a, b) \in R$ and $(b, c) \in R$."

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- Use the conditions for both pairs to logically or algebraically prove that the condition must hold for the pair (a, c).
- Conclude: "Therefore, (a, c) ∈ R, and R is transitive."

- **Step 4: Final Conclusion** (Conclude)
  - State: "Since R is reflexive, symmetric, and transitive, R is an equivalence relation."

- **Variants:** If asked to *disprove*, you only need to find a single counterexample for any one of the three properties.
- **When NOT to Use:** This formal proof method is overkill if the set A is small and the relation is given as a list of pairs (roster method). In that case, you can check by direct inspection.

3.3 How to Write Answers

Scoring full marks isn't just about getting the right answer; it's about presenting it in a way that leaves no room for doubt. Use this foolproof structure for your proofs.

- **Answer Template: Equivalence Relation Proof Frame**
  - **When to Use:** Use this precise structure for any question asking you to "Show that..." or "Prove that..." a relation is an equivalence relation.
  - **Line-by-Line Structure:**
    - L1: **Reflexivity:**
    - L2: State the condition for (a, a) and show it's true for any a in the given set. Conclude that R is reflexive.
    - L3: **Symmetry:**
    - L4: State "Assume (a, b) ∈ R." Write what this implies based on the relation's definition.
    - L5: Logically manipulate the implication to show that the condition for (b, a) must also be true. Conclude that R is symmetric.
    - L6: **Transitivity:**
    - L7: State "Assume (a, b) ∈ R and (b, c) ∈ R." Write the two implications from this assumption.
    - L8: Combine the two implications to prove that the condition for (a, c) must be true. Conclude that R is transitive.
    - L9: **Conclusion:**

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- L10: Write the final sentence: "Since R is reflexive, symmetric, and transitive, it is an equivalence relation."

- **Essential Phrases:** "For any a ∈ A...", "Let (a, b) ∈ R, which implies...", "Therefore, ...", "Hence, R is reflexive/symmetric/transitive."
- **General Rules:**

1. Always handle each property under its own clear heading.
2. Start your symmetry and transitivity proofs with a clear assumption ("Let...").
3. Never skip the final concluding sentence that summarizes the result.

3.4 Critical Checks and Common Mistakes

Pay close attention to this section. These are the most common traps where students lose marks. Here's how to avoid them.

A. Foundational Logic: The "For Every" Rule in Reflexivity

- **Rule:** The condition for reflexivity, (a, a) ∈ R, must be true for **every single element** a in the set A, without exception.
- **When:** This is the very first check. If you can find even one element 'a' for which (a, a) is not in R, the relation is not reflexive, and you can stop (it cannot be an equivalence relation).
- **Linked:** This is the foundation of the Reflexivity step in the proof method.

B. Proof-Writing Pitfalls

- **Pitfall 1: Incomplete Reflexivity Check**
  - **Category:** Logic
  - **Occurs In:** Checking for Reflexivity (Step 1)
  - **Wrong:** Picking one example element (like 1) and showing (1, 1) is in R, then assuming it's reflexive.
  - ✓ **Fix:** The proof must work for an *arbitrary* element 'a' from the set A. Always start with "Let a ∈ A..." to show it holds for all elements. This directly follows the "For Every" rule.
- **Pitfall 2: Circular Logic in Symmetry**
  - **Category:** Logic
  - **Occurs In:** Checking for Symmetry (Step 2)

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  - **Wrong:** Assuming (a, b) ∈ R and (b, a) ∈ R at the same time. You cannot assume the conclusion.
  - ✓ **Fix:** You must *assume only* that (a, b) ∈ R is true, and then use that fact to *prove* that (b, a) ∈ R must follow.
- **Pitfall 3: Broken Chain in Transitivity**
  - **Category:** Logic
  - **Occurs In:** Checking for Transitivity (Step 3)
  - **Wrong:** Just saying "If (1, 2) and (2, 3) are in R, then (1, 3) is in R" without showing *why* based on the relation's definition.
  - ✓ **Fix:** You must use the actual conditions. For example, if R is "a-b is even", you must show that (a-b) + (b-c) = a-c, and since the sum of two even numbers is even, the condition holds for (a, c).

3.5 Exam Strategy

Focus your preparation on these key areas to maximize your score.

- **Example Range:** Master the concepts in NCERT Examples 1-6 (Pages 2-4). Pay special attention to Example 3 (perpendicular lines) and Example 5 (divisibility) as they cover key ideas.
- **Exercise Sets:** Focus on Exercise 1.1, Questions 1-14. These questions provide comprehensive practice on all problem types.
- **Question Patterns:** Be prepared for three main types of questions:
  1. Check if a given relation has one or more properties (reflexive, symmetric, transitive).
  2. Prove that a given relation is an equivalence relation (a full, three-part proof).
  3. Describe the equivalence classes for a proven equivalence relation (like the sets of even and odd integers).
- **Approach:** First, master checking each of the three properties (R, S, T) individually using simple examples. Once you are confident, move on to writing full, structured proofs for equivalence relations.

3.6 Topic Connections

Understanding how this topic fits with others gives you a bigger picture of the subject.

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- **Prerequisites:** This topic builds directly on your Class 11 knowledge of **Set Theory** (relations are defined on sets) and **Cartesian Products** (a relation R in set A is always a subset of A × A). A solid understanding of these is essential.
- **Forward Links:** Understanding types of relations, especially equivalence relations, is critical for the concept of **Equivalence Classes**, which is about partitioning a set into groups of related elements. It also lays the groundwork for **Functions**, as functions are a special, more restrictive type of relation.

3.7 Revision Summary

This is a quick summary of the key definitions from the NCERT textbook, perfect for last-minute revision.

- **Key Points for Revision:**
  - **Empty relation** is the relation R in X given by $R = \emptyset \subset X \times X$.
  - **Universal relation** is the relation R in X given by $R = X \times X$.
  - **Reflexive relation** R in X is a relation with $(a, a) \in R$ for all $a \in X$.
  - **Symmetric relation** R in X is a relation satisfying $(a, b) \in R$ implies $(b, a) \in R$.
  - **Transitive relation** R in X is a relation satisfying $(a, b) \in R$ and $(b, c) \in R$ implies that $(a, c) \in R$.
  - **Equivalence relation** R in X is a relation which is reflexive, symmetric, and transitive.
  - **Equivalence class** [a] containing $a \in X$ for an equivalence relation R in X is the subset of X containing all elements b related to a.

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